P.G.-sem-2nd
paper-VI, unit-1st-
complex integration
Dr. G.D. Singh

26 बुध

③ Obtain the expression for $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ which are valid when (i) $|z|<1$ (ii) $1<|z|<4$
(iii) $|z|>4$

Here $f(z) = \frac{z^2-4}{z^2+5z+4}$

$\therefore \; z^2+5z+4 \overline{)\, z^2-4 \,}(\, 1$
$\quad \underline{z^2 \pm 5z \pm 4}$
$\quad -5z-8$

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		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19

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27 गुरु

$$\therefore f(z) = 1 - \frac{5z+8}{z^2+5z+4}$$

$$= 1 - \frac{5z+8}{(z+1)(z+4)} \quad \text{---} \; (1)$$

Now $\dfrac{5z+8}{(z+1)(z+4)} = \dfrac{A}{z+1} + \dfrac{B}{z+4}$

$$\therefore 5z+8 = A(z+4) + B(z+1)$$

$z=-4$, $\quad 5(-4)+8 = A(0) + B(-4+1)$

$$-20+8 = -3B \quad \therefore B = \frac{-12}{-3} = 4$$

$z=-1$, $\therefore -5+8 = A(-1+4) + B(0)$

28 शुक्र

$$+3 = 3A \quad \therefore A = 1$$

$\therefore$ from (1)

$$f(z) = 1 - \left\{ \frac{1}{z+1} + \frac{4}{z+4} \right\}$$

$$= 1 - \frac{1}{1+z} - \frac{4}{4+z}$$

(i.) when $|z| < 1$

So $f(z) = 1 - \dfrac{1}{1+z} - \dfrac{4}{4\left(1+\frac{z}{4}\right)}$

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19	20	21	22	23	24	25

$$= 1-(1+z)^{-1}-\left(1+\frac{z}{4}\right)^{-1}$$

$$= 1-\left\{1-z+z^2-z^3+\cdots+(-1)^n z^n+\cdots\right\}$$

$$-\left\{1-\frac{z}{4}+\left(\frac{z}{4}\right)^2-\left(\frac{z}{4}\right)^3+\cdots(-1)^n\left(\frac{z}{4}\right)^n+\cdots\right\}$$

$$= 1-1+z-z^2+z^3-\cdots-(-1)^n z^n-\cdots$$

$$-1+\frac{z}{4}-\left(\frac{z}{4}\right)^2+\left(\frac{z}{4}\right)^3-\cdots-(-1)^n\frac{z^n}{4^n}-\cdots$$

$$= -1+\sum_{n=1}^{\infty}(-1)^{n+1}\left\{1+\frac{1}{4^n}\right\}z^n \quad \text{Ans}$$

This is Maclaurent's Series

$$\left[-(-1)^n = (-1)^1(-1)^n = (-1)^{n+1}\right.$$